SURFACE AREA AND VOLUME OF SOLIDS

Geometry

Chapter 12

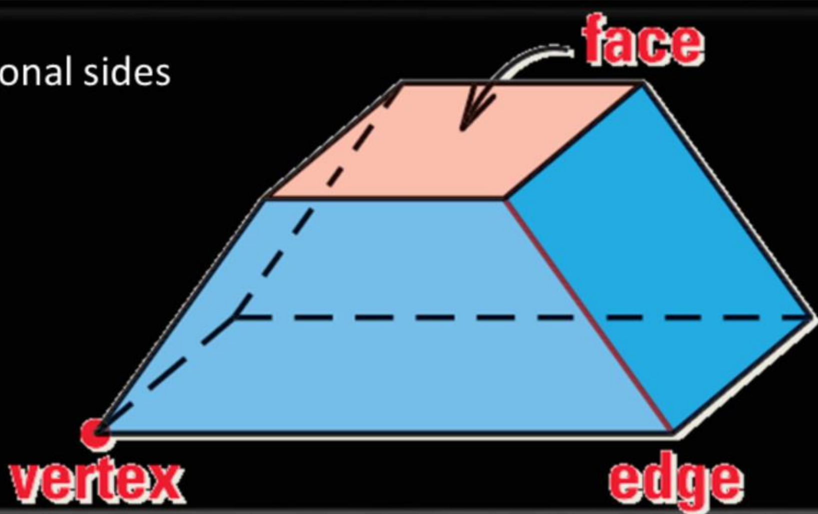
Geometry 12

- This Slideshow was developed to accompany the textbook
  - *Larson Geometry*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

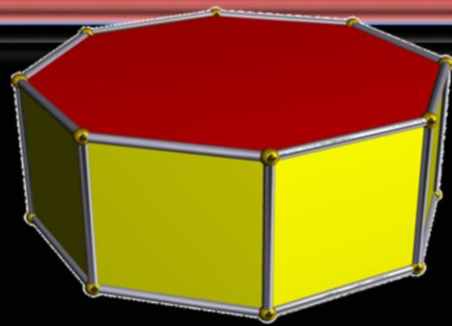
Slides created by
Richard Wright, Andrews Academy
rwright@andrews.edu

12.1 EXPLORE SOLIDS

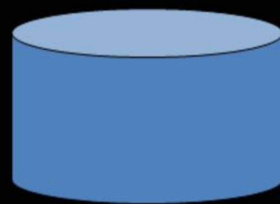
- Polyhedron
  - Solid with polygonal sides
  - Flat sides
- Face
  - Side
- Edge
  - Line segment
- Vertex
  - Corner



12.1 EXPLORE SOLIDS

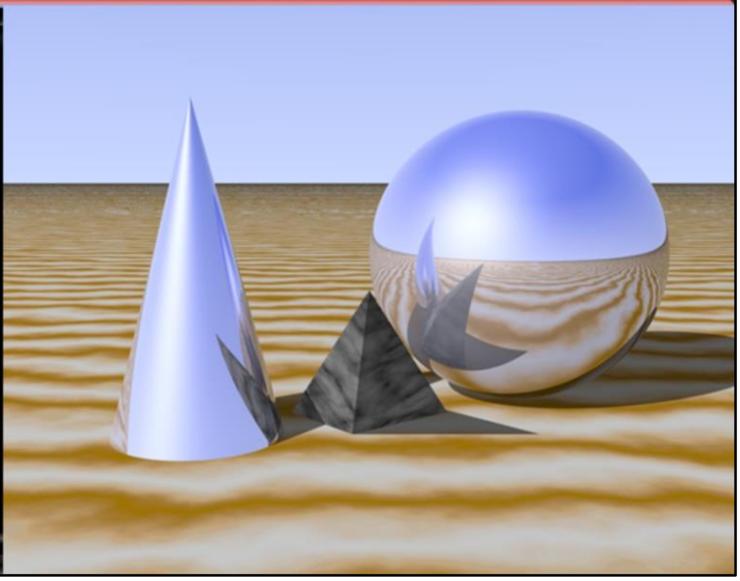


- Prism
  - Polyhedron with two congruent surfaces on parallel planes (the 2 ends (**bases**) are the same)
  - Named by bases (i.e. rectangular prism, triangular prism)
- Cylinder
  - Solid with congruent circular bases on parallel planes (not a polyhedron)



12.1 EXPLORE SOLIDS

- Pyramid → polyhedron with all but one face intersecting in one point
- Cone → circular base with the other surface meeting in a point (kind of like a pyramid)
- Sphere → all the points that are a given distance from the center



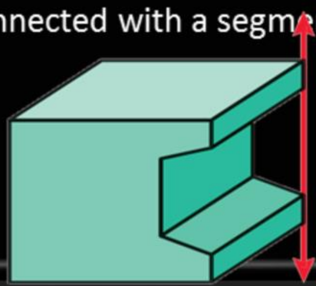
12.1 EXPLORE SOLIDS

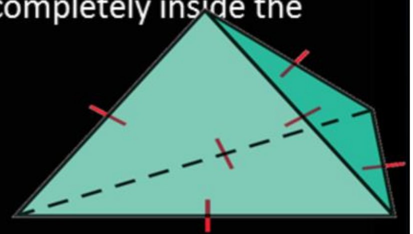
Euler's Theorem

The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by

$$F + V = E + 2$$

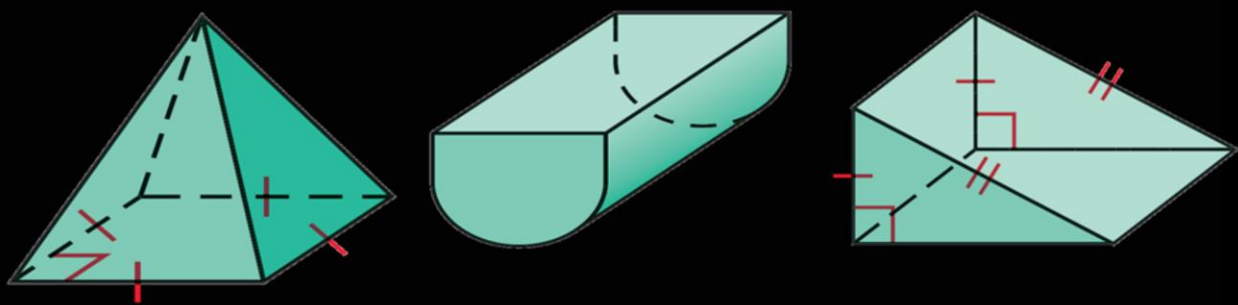
- Convex
  - Any two points can be connected with a segment completely inside the polyhedron
- Concave
  - Not convex
  - Has a "cave"





12.1 EXPLORE SOLIDS

- Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges and describe as convex or concave

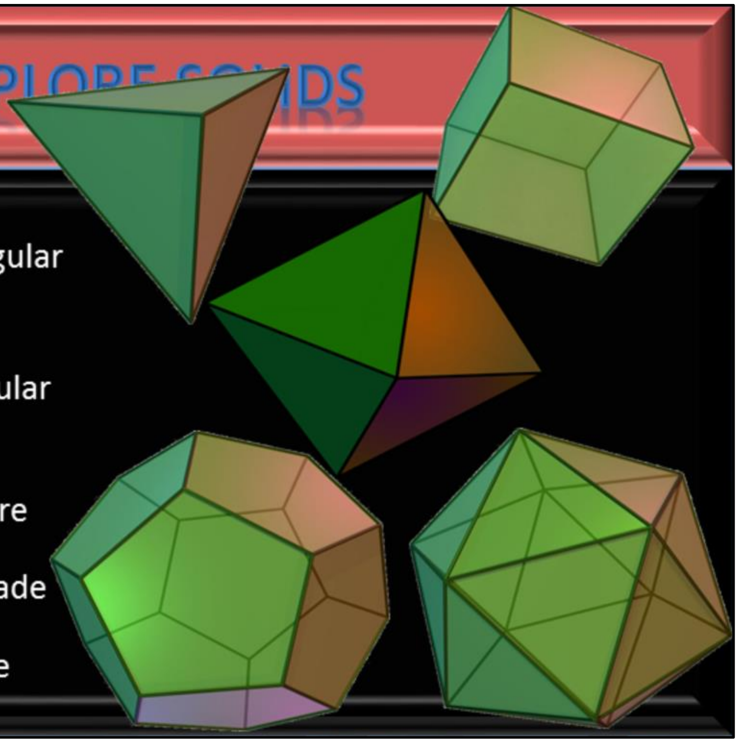


Polyhedron; Square Pyramid; 5 faces, 5 vertices, 8 edges; convex

Not a Polyhedron

Polyhedron; Triangular Prism; 5 faces, 6 vertices, 9 edges; convex

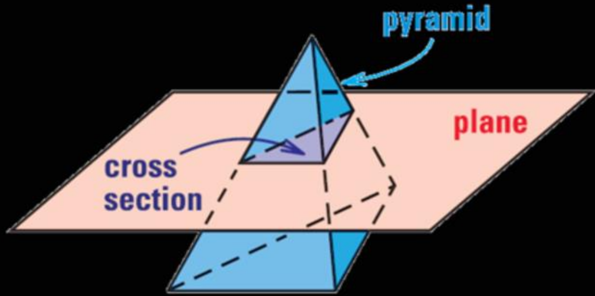
12.1 EXPLORE SOLIDS



- Regular polyhedron
  - Polyhedron with congruent regular polygonal faces
- Only 5 types (**Platonic solids**)
  - **Tetrahedron** → 4 faces (triangular pyramid)
  - **Hexahedron** → 6 faces (cube)
  - **Octahedron** → 8 faces (2 square pyramids put together)
  - **Dodecahedron** → 12 faces (made with pentagons)
  - **Icosahedron** → 20 faces (made with triangles)

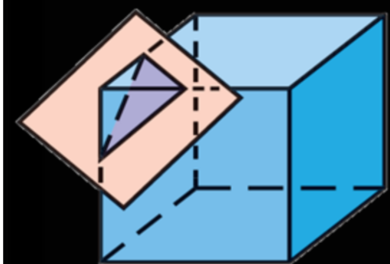
12.1 EXPLORE SOLIDS

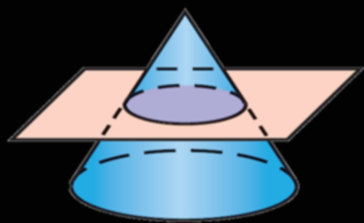
- Cross Section
  - ◇Imagine slicing a very thin slice of the solid
  - ◇The cross section is the 2-D shape of the thin slice

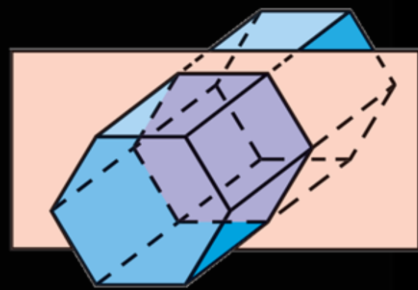


12.1 EXPLORE SOLIDS

- Find the number of faces, vertices, and edges of a regular dodecahedron. Check with Euler's Theorem.
- Describe the cross section.







- *798 #2-40 even, 44-60 even = 29*

Triangle

Circle

Hexagon

ANSWERS AND QUIZ

- 12.1 Answers
- 12.1 Homework Quiz

12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- Surface area = sum of the areas of each surface of the solid
  - In order to calculate surface area it is sometimes easier to draw all the surfaces

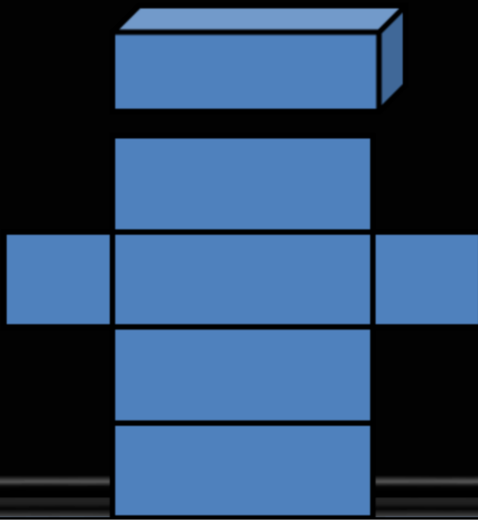
Some sports relie on having very little friction. In biking for example, the smaller the surface area of the tires, the less friction there is. And thus the faster the rider can go.

→ Draw the top triangle first (for some triangles you may have to count a horizontal space as 2)

12.2 SURFACE AREA OF PRISMS AND CYLINDERS

Nets

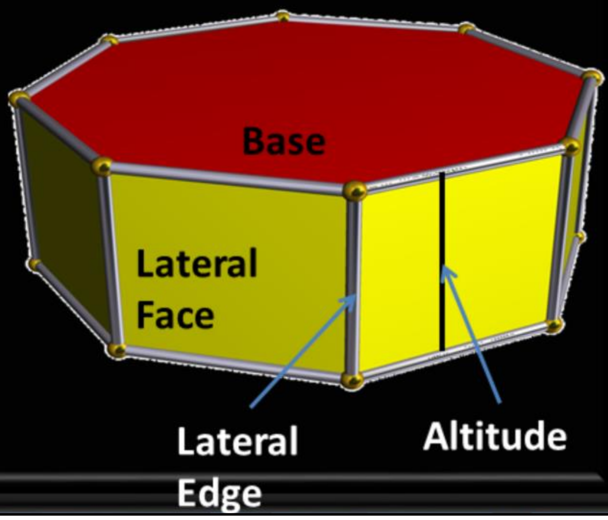
- Imagine cutting the three dimensional figure along the edges and folding it out.
- Start by drawing one surface, then visualize unfolding the solid.
- To find the surface area, add up the area of each of the surfaces of the net.



12.2 SURFACE AREA OF PRISMS AND CYLINDERS

Parts of a right prism

- Bases → parallel congruent surfaces (the ends)
- Lateral faces → the other faces (they are parallelograms)
- Lateral edges → intersections of the lateral faces (they are parallel)
- Altitude → segment perpendicular planes containing the two bases with an endpoint on each plane
- Height → length of the altitude



12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- Right prism
  - Prism where the lateral edges are altitudes

- Oblique prism
  - Prism that isn’t a right prism

12.2 SURFACE AREA OF PRISMS AND CYLINDERS

Lateral Area (L) of Prisms

- Area of the Lateral Faces
- L = Ph
  - L = Lateral Area
  - P = Perimeter of base
  - h = Height

You can find the surface area by adding up the areas of each surface, but if you could use a formula, it would be quicker

All the lateral surfaces are rectangles

Area = bh

Add up the areas L = ah + bh + ch + … + dh

L = (a + b + c + … + d)h

Perimeter of base = a + b + c + … + d

L = Ph

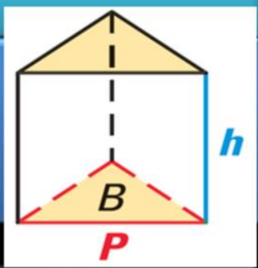
12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- Base Area (B)
  - ◇In a prism, both bases are congruent, so you only need to find the area of one base and multiply by two

Surface Area of a Right Prism

$$S = 2B + Ph$$

Where S = surface area, B = base area, P = perimeter of base, h = height of prism



12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- Draw a net for a triangular prism.

- Find the lateral area and surface area of a right rectangular prism with height 7 inches, length 3 inches, and width 4 inches.

$$P = 2(3) + 2(4) = 14$$
$$L = (14)(7) = 98$$
$$B = 3 \cdot 4 = 12$$
$$A = 2(12) + 14(7) = 122$$

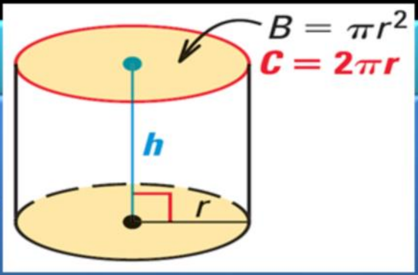
12.2 Surface Area of Prisms and Cylinders

- Cylinders are the same as prisms except the bases are circles
  - ◇Lateral Area = L = 2πrh

Surface Area of a Right Cylinder

$$S = 2\pi r^2 + 2\pi rh$$

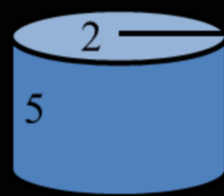
Where S = surface area, r = radius of base, h = height of prism



12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- The surface area of a right cylinder is 100 cm^2. If the height is 5 cm, find the radius of the base.

- Example: Draw a net for the cylinder and find its surface area.



- *806 #2-28 even, 31-37 all = 21*

$$100 = 2\pi r^2 + 2\pi r(5)$$
$$100 = 2\pi r^2 + 10\pi r$$
$$0 = 2\pi r^2 + 10\pi r - 100$$
$$0 = r^2 + 5r - 15.915$$
$$r = \frac{-5 \pm \sqrt{5^2 - 4(1)(-15.915)}}{2(1)}$$
$$r = \frac{-5 \pm \sqrt{88.662}}{2}$$
$$r = 2.2, -7.2$$

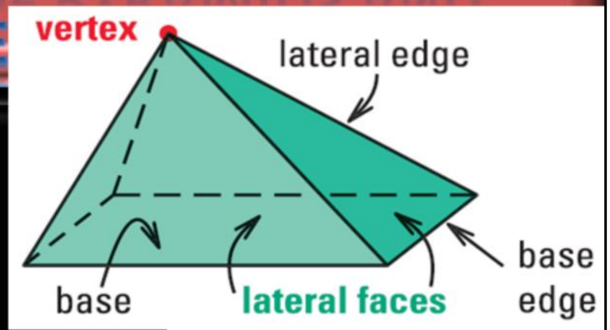
Only 2.2 makes sense because the radius must be positive

$$S = 2\pi 2^2 + 2\pi(2)(5)$$
$$S = 8\pi + 20\pi = 28\pi$$

ANSWERS AND QUIZ

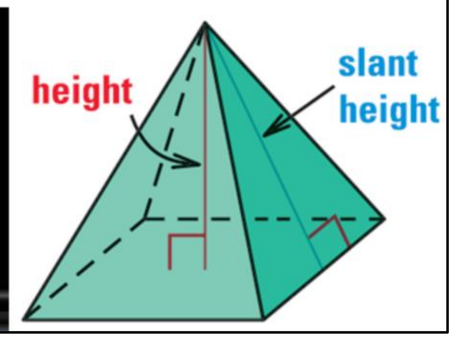
- 12.2 Answers
- 12.2 Homework Quiz

12.3 SURFACE AREA OF PYRAMIDS AND CONE



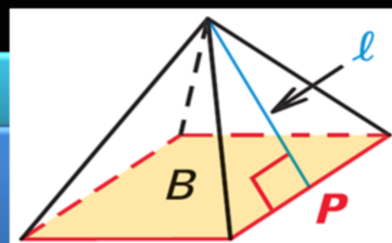
Pyramids

- All faces except one intersect at one point called **vertex**
- The **base** is the face that does not intersect at the vertex
- **Lateral faces** → faces that meet in the vertex
- **Lateral edges** → edges that meet in the vertex
- **Altitude** → segment that goes from the vertex and is perpendicular to the base



12.3 SURFACE AREA OF PYRAMIDS AND CONES

- **Regular pyramid** → base is a regular polygon and the vertex is directly above the center of the base
  - ◇ In a regular pyramid, all the lateral faces are congruent isosceles triangles
  - ◇ The height of each lateral face is called the **slant height** (ℓ)
- **Lateral Area** → L = ½ Pℓ



Surface Area of a Regular Pyramid

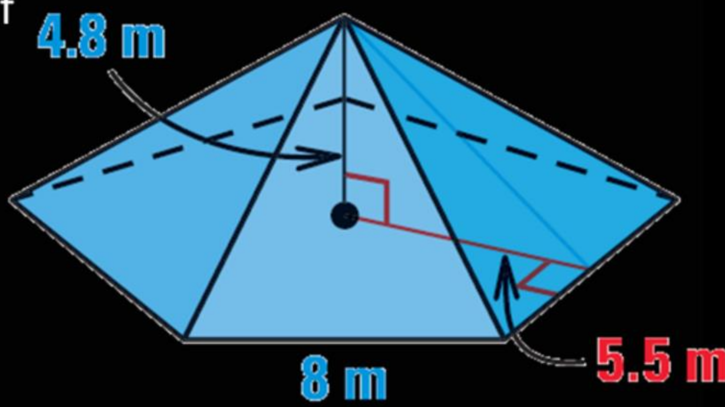
$$S = B + \frac{1}{2}P\ell$$

Where B = base area, P = base perimeter, ℓ = slant height

Lateral area is ½ because the sides are triangles.

12.3 SURFACE AREA OF PYRAMIDS AND CONES

- Find the surface area of the regular pentagonal pyramid.



Base Area

$$B = \frac{1}{2}Pa$$

$$B = \frac{1}{2}(5 \cdot 8)(5.5) = 110$$

$$\ell^2 = 5.5^2 + 4.8^2$$

$$\ell = 7.3$$

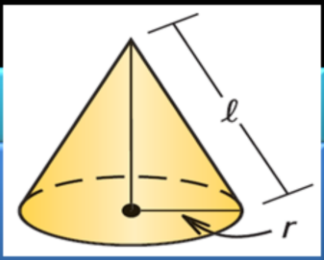
$$S = B + \frac{1}{2}P\ell$$

$$S = 110 + \frac{1}{2}(5 \cdot 8)(7.3) = 256$$

12.3 SURFACE AREA OF PYRAMIDS AND CONES

Cones

- Cones are just like pyramids except the base is a circle
- Lateral Area = $\pi r \ell$



Surface Area of a Right Cone

$$S = \pi r^2 + \pi r \ell$$

Where r = base radius, ℓ = slant height

12.3 SURFACE AREA OF PYRAMIDS AND CONES

- Example: The So-Good Ice Cream Company makes Cluster Cones. For packaging, they must cover each cone with paper. If the diameter of the top of each cone is 6 cm and its slant height is 15 cm, what is the area of the paper necessary to cover one cone?

- *814 #2-32 even, 35-39 all = 21*
- *Extra Credit 817 #2, 6 = +2*

Looking for lateral area.

$$L = \pi 3(15) = 141.4\ cm^2$$

ANSWERS AND QUIZ

- 12.3 Answers
- 12.3 Homework Quiz

12.4 VOLUME OF PRISMS AND CYLINDERS

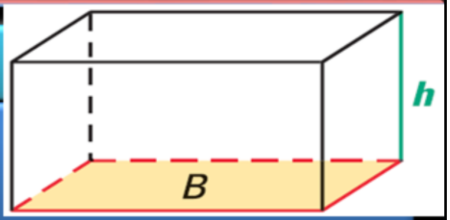
- Create a right prism using geometry cubes
- Count the lengths of the sides
- Count the number of cubes.
- Remember this to verify the formulas we are learning today.

12.4 VOLUME OF PRISMS AND CYLINDERS

Volume of a Prism

$$V = Bh$$

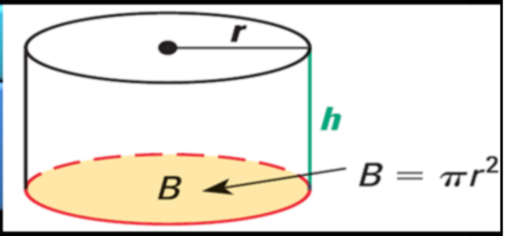
Where B = base area, h = height of prism



Volume of a Cylinder

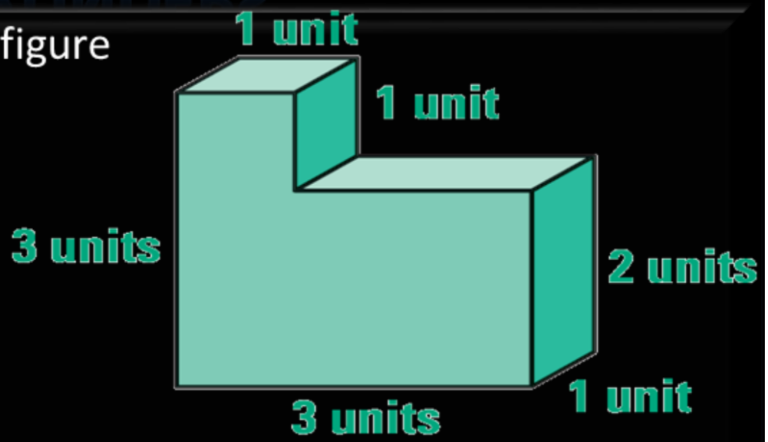
$$V = \pi r^2 h$$

Where r = radius, h = height of cylinder



12.4 VOLUME OF PRISMS AND CYLINDERS

- Find the volume of the figure



Cut into two prisms

Top

$$V = 1(1)(1) = 1$$

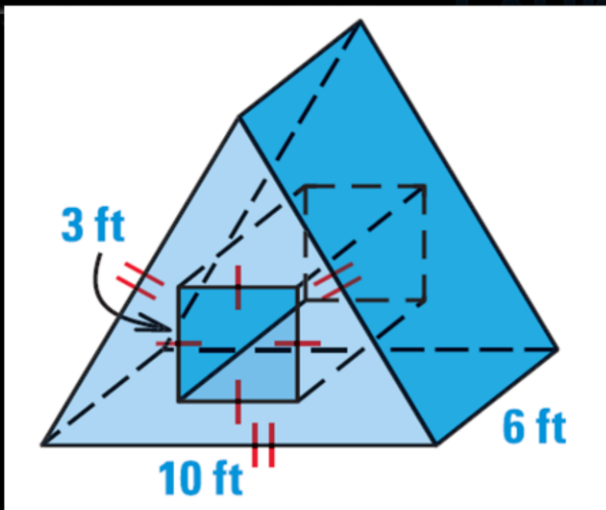
Bottom

$$V = 3(1)(2) = 6$$

Total

$$V = 1 + 6 = 7$$

12.4 VOLUME OF PRISMS AND CYLINDERS



- Find the volume.

Base Area (front)
Find height of triangle

$$5^2 + x^2 = 10^2$$
$$25 + x^2 = 100$$
$$x^2 = 75$$
$$x = 5\sqrt{3}$$

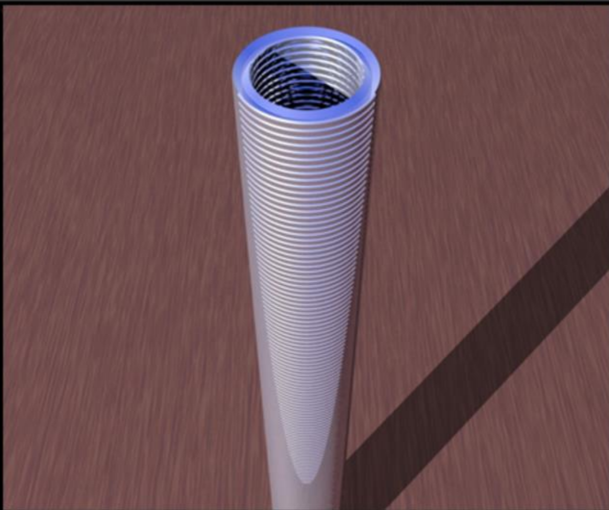
Area=triangle - square

$$B = \frac{1}{2}(10)\left(5\sqrt{3}\right) - 3^2$$
$$B = 25\sqrt{3} - 9 \approx 34.301$$

Volume = Bh

$$V = \left(25\sqrt{3} - 9\right)(6) = 150\sqrt{s} - 54 \approx 205.8$$

12.4 VOLUME OF PRISMS AND CYLINDERS



- There are 150 1-inch washers in a box. When the washers are stacked, they measure 9 inches in height. If the inside hole of each washer has a diameter of ¾ inch, find the volume of metal in one washer.

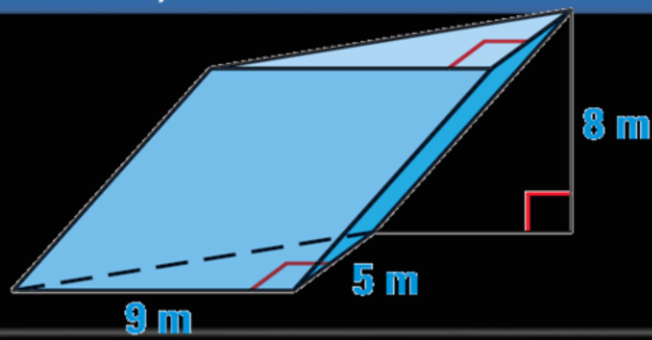
Find volume of washers without holes: $V = \pi \, \tfrac{1}{2}^2 \, 9 = 7.06858$
Find volume of hole: $V = \pi(3/8)^2 \, 9 = 3.97608$
Find volume of washers with holes: $7.06858 - 3.97608 = 3.09251$
Find volume of one washer: $3.09251/150 = 0.02 \text{ in}^3$

12.4 VOLUME OF PRISMS AND CYLINDERS

Cavalieri's Principle

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

- Find the volume.



- *822 #2-40 even = 20*

$$B = \frac{1}{2}(9)(5) = 22.5\ m^2$$
$$V = (22.5\ m^2)(8\ m) = 180\ m^3$$

ANSWERS AND QUIZ

- 12. 4 Answers
- 12.4 Homework Quiz

12.5 VOLUME OF PYRAMIDS AND CONES

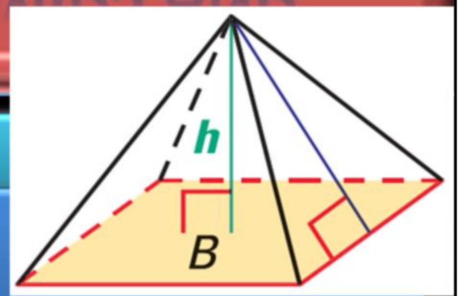
- How much ice cream will fill an ice cream cone?
- How could you find out without filling it with ice cream?
- What will you measure?

12.5 VOLUME OF PYRAMIDS AND CONES

Volume of a Pyramid

$$V = \frac{1}{3}Bh$$

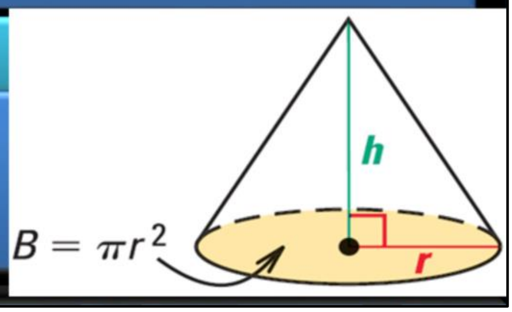
Where B = base area, h = height of pyramid



Volume of a Cone

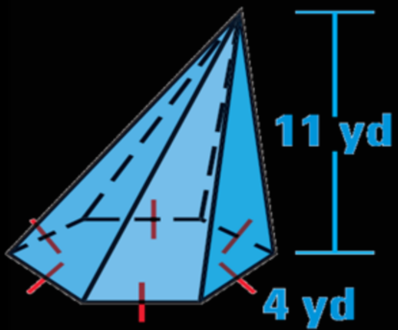
$$V = \frac{1}{3}\pi r^2 h$$

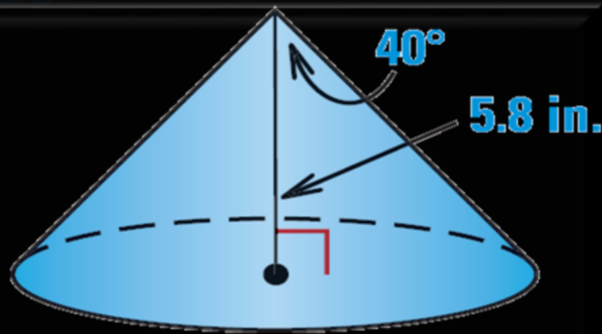
Where r = radius, h = height of cone



12.5 Volume of Pyramids and Cones

- Find the volume.





- *832 #2-30 even, 34, 36, 40, 44-52 even = 23*
- *Extra Credit 836 #2, 4 = +2*

$$B = \frac{1}{2}Pa$$

$$\frac{1}{2}central\ angle = \frac{1}{2}\left(\frac{360}{6}\right) = 30$$

$$\tan 30 = \frac{2}{a}$$

$$a = \frac{2}{\tan 30} = 3.464$$

$$B = \frac{1}{2}(4 \cdot 6)(3.464) = 41.569$$

$$V = \frac{1}{3}(41.569)(11) = 152.42$$

$$\tan 40 = \frac{r}{5.8}$$

$$r = 5.8 \cdot \tan 40 = 4.8668$$

$$V = \frac{1}{3}\pi 4.8668^2 \cdot 5.8 = 143.86$$

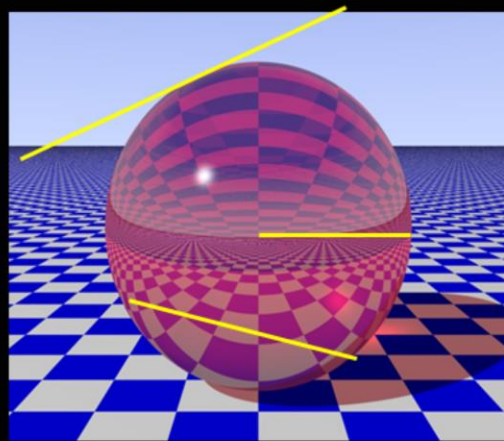
ANSWERS AND QUIZ

- 12.5 Answers
- 12.5 Homework Quiz

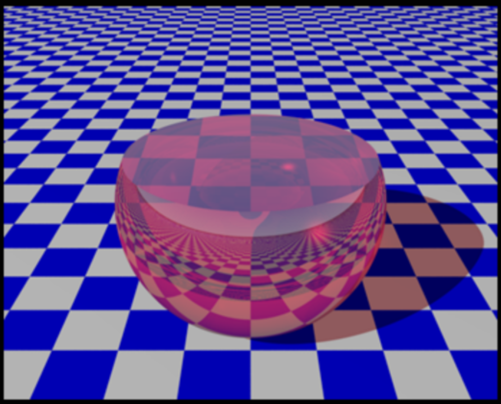
12.6 SURFACE AREA AND VOLUME OF SPHERES

Terms

- **Sphere** → all points equidistant from center
- **Radius** → segment from center to surface
- **Chord** → segment that connects two points on the sphere
- **Diameter** → chord contains the center of the sphere
- **Tangent** → line that intersects the sphere in exactly one place



12.6 SURFACE AREA AND VOLUME OF SPHERES



- Intersections of plane and sphere
  - Point → plane tangent to sphere
  - Circle → plane not tangent to sphere
  - Great Circle → plane goes through center of sphere (like equator)
    - Shortest distance between two points on sphere
    - Cuts sphere into two **hemispheres**

12.6 SURFACE AREA AND VO[illegible]E OF SPHERES

Surface Area of a Sphere

$$S = 4\pi r^2$$

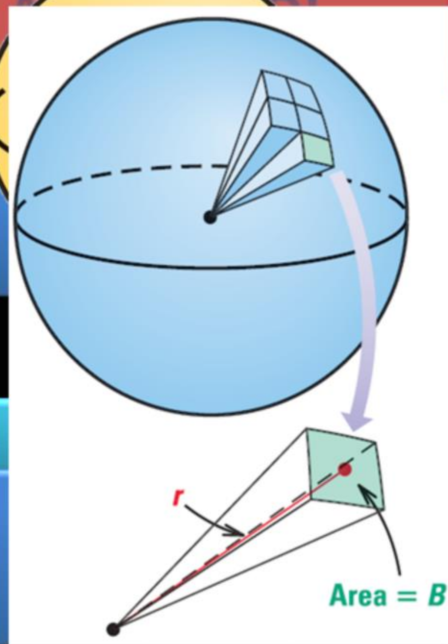
Where r = radius

- If you cut 4 circles into 8ths you can put them together to make a sphere

Volume of a Sphere

$$V = \frac{4}{3}\pi r^3$$

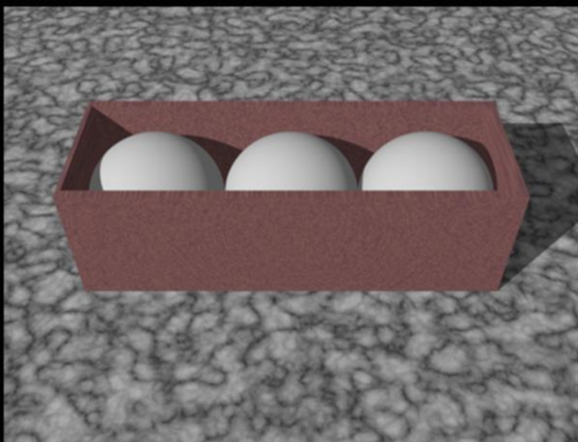
Where r = radius



You can think about cutting a sphere into many small regular square pyramids.

V = 1/3 Bh → the area of all the bases is $4\pi r^2$ and h = r

12.6 SURFACE AREA AND VOLUME OF SPHERES



- Find the volume of the empty space in a box containing three golf balls. The diameter of each is about 1.5 inches. The box is 4.5 inches by 1.5 inches by 1.5 inches.
- *842 #2-36 even, 40-44 even = 21*

Volume of box: $4.5(1.5)(1.5) = 10.125$
Volume of each ball: $\frac{4}{3}\pi.75^3 = 1.767$
Volume of empty space: $10.125 - 3(1.767) = 4.824$

ANSWERS AND QUIZ

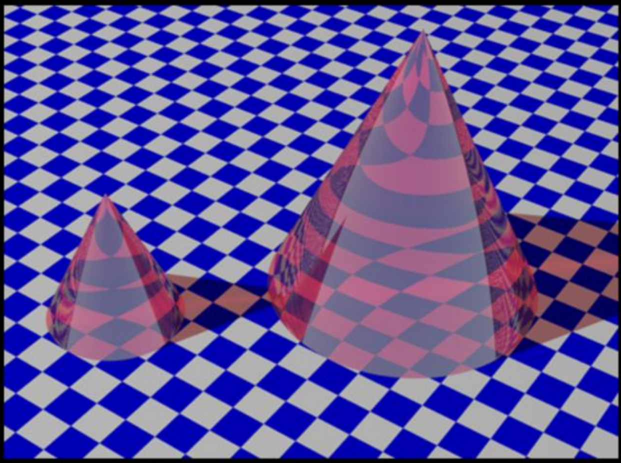
- 12.6 Answers

- 12.6 Homework Quiz

12.7 EXPLORE SIMILAR SOLIDS

- Russian Matryoshka dolls nest inside each other. Each doll is the same shape, only smaller. The dolls are similar solids.

12.7 EXPLORE SIMILAR SOLIDS



- Similar Solids
  - Solids with same shape but not necessarily the same size
  - The lengths of sides are proportional
  - The ratios of lengths is called the scale factor

12.7 EXPLORE SIMILAR SOLIDS

- Congruent Solids
  - Similar solids with scale factor of 1:1
- Following four conditions must be true
  - Corresponding angles are congruent
  - Corresponding edges are congruent
  - Areas of corresponding faces are equal
  - The volumes are equal

12.7 EXPLORE SIMILAR SOLIDS

- Determine if the following pair of shapes are similar, congruent or neither.
  - Cone A: r = 4.3, h = 12, slant height = 14.3
  - Cone B: r = 8.6, h = 25, slant height = 26.4
    - Ratios: $\frac{8.6}{4.3} = 2, \frac{25}{12} = 2.08$. Not proportional so neither
  - Right Cylinder A: r = 5.5, height = 7.3
  - Right Cylinder B: r = 5.5, height = 7.3
    - 1:1 ratio so congruent.

12.7 EXPLORE SIMILAR SOLIDS

Similar Solids Theorem

If 2 solids are similar with a scale factor of a:b,
then the areas have a ratio of $a^2:b^2$
and the volumes have a ratio of $a^3:b^3$

12.7 EXPLORE SIMILAR SOLIDS

- Cube C has a surface area of 216 square units and Cube D has a surface area of 600 square units. Find the scale factor of C to D.
- Find the edge length of C.
- Use the scale factor to find the volume of D.
- *850 #2-26 even, 30-48 even = 23*
- *Extra Credit 854 #2, 4 = +2*

Areas: $\frac{216}{600} = \frac{9}{25} = \frac{c^2}{d^2}$

$\frac{c}{d} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$

Cube surface area: $S = 6c^2$

$216 = 6c^2$

$36 = c^2$

$c = 6$

Volumes: $\frac{c^3}{d^3} = \frac{3^3}{5^3}$

$\frac{6^3}{d^3} = \frac{3^3}{5^3}$

$\frac{216}{d^3} = \frac{27}{125}$

$27d^3 = 216(125)$

$d^3 = 1000$

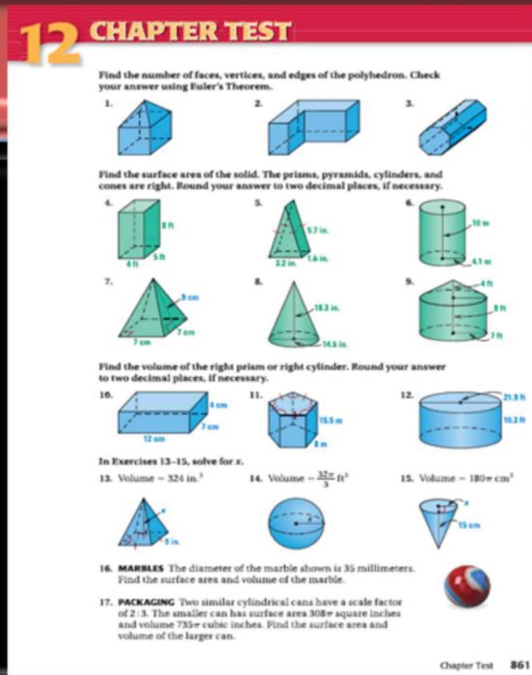
volume of D is 1000

ANSWERS AND QUIZ

- 12.7 Answers
- 12.7 Homework Quiz

12.REVIEW

- *861 #1-17 all = 17*



12 CHAPTER TEST

Find the number of faces, vertices, and edges of the polyhedron. Check your answer using Euler's Theorem.

1.

2.

3.

Find the surface area of the solid. The prisms, pyramids, cylinders, and cones are right. Round your answer to two decimal places, if necessary.

4.

5.

6.

7.

8.

9.

Find the volume of the right prism or right cylinder. Round your answer to two decimal places, if necessary.

10.

11.

12.

In Exercises 13–15, solve for x.

13. Volume $= 324$ in.^{3}

14. Volume $= \frac{32\pi}{3}$ ft^3

15. Volume $= 180\pi$ cm^3

16. **MARBLES** The diameter of the marble shown is 35 millimeters. Find the surface area and volume of the marble.

17. **PACKAGING** Two similar cylindrical cans have a scale factor of 2 : 3. The smaller can has surface area 308π square inches and volume 735π cubic inches. Find the surface area and volume of the larger can.

Chapter Test 861